

(1)

Electronics basic Concepts:

Definitions, fundamental laws and theorems

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Basic concepts of electronics:

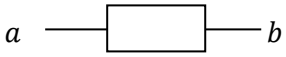
Definitions, fundamental laws and theorems

A. Definitions

I. Electrical circuit: Definitions

An electrical circuit is a set of electrical components interconnected in any way by conductors (wires).

An electrical component is an element with 2 terminals (this is called a dipole. Examples: resistor, capacitor, coil) or several terminals (3 in the case of a transistor, for example, 4 in the case of a quadrupole ...). A dipole is usually represented in the following form:



Terminals a and b are used to connect with the other components.

A conductor is formed of a material that carries electric current well (example: Copper). It is most often a metal, but it is also possible to use a conductive liquid. This is called an electrolyte.

II. Currents, voltage, power:

1. Current

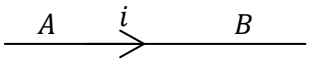
An electric current is an orderly displacement of electrical charges in a conductor. It is characterized by a quantity, the intensity, defined as the flow of electrical charges in the conductor

This magnitude is often denoted *i*. When, for a time *dt*, it passes *dq* Coulombs, the intensity is worth:

$$i(t) = \frac{dq}{dt}$$

The legal unit in which the intensity of the electric current is expressed is the ampere (symbol A). An intensity of 1 A corresponds to a flow rate of 1 coulomb per second.

The current in the diagram of an electrical circuit is represented by an arrow on a conductor (the current flows through a component).



The noted current *i* is marked in an arbitrary direction. This does not mean that the current flows from A to B but that, if the result of the calculation gives a positive numerical value, the current flows well from A to B, otherwise it flows in the opposite direction.

It should be noted that due to the definition of the intensity ($i(t) = \frac{dq}{dt}$) and the charge of the electron (negative charge), the direction of effective movement of electrons is the opposite of the conventional positive direction of current. This small inconsistency has historical origins, the electron having been discovered after the identification and formalization of the electrical phenomenon.

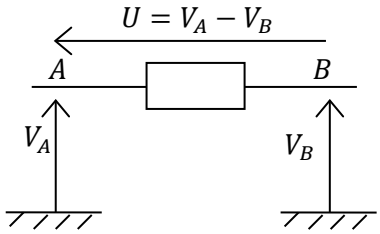
1. Potential difference: voltage

At rest, the electrical charges in a conductor are in continuous motion under the effect of thermal agitation.

However, this movement does not translate into an overall displacement that can translate into electric current. To set these charges in motion in a given direction, it is necessary to apply an electric field $\vec{E}$ at the conductor's terminals. By applying the electric potentials V_1 and V_2 at its 2 terminals, we create a potential difference that puts the electrons in an oriented and ordered motion, because they are then subjected to the electric force $\vec{F} = q.\vec{E}$.

The value of the potential difference (often abbreviated as "p.d.") is called the voltage, and its unit is the Volt (symbol V). The Volt is defined in such a way that a charge of an accelerated Coulomb under a voltage of 1V acquires an energy of $1J : 1V = 1\,J/C$.

We represent a potential difference by an arrow next to the component (the voltage is a potential difference between the 2 terminals of the component), as on the following diagram.



By convention, the voltage noted U represents the potential difference $V_A - V_B$ (arrowhead – back of arrow). So, if, after numeric application, the voltage U is positive, this means that the potential of the point A is greater than that of the point B, and if U is negative this means that the potential of the point A is less than that of the point B.

At the bottom of this diagram, the crossed-out symbols indicate the potential reference chosen as zero, called the ground, with respect to which the potentials V_A and V_B are defined. We then sometimes talk about the voltage at one point.

BE CAREFUL not to confuse ground and earth: the earth is the POINT of zero potential, while the ground is a point that is taken as a reference.

III. Dipoles and sign conventions

1. Definitions

A dipole is an element with 2 terminals (or poles): it is passed by a current. The same current is flowing between the two poles. Each of these 2 poles has a certain electrical potential and the resulting potential difference is the voltage at dipole terminals.

A dipole can be linear or non-linear, passive or active, receiver or source.

Let's define these different terms.

- A dipole is **linear** if the function f, such that $U = f(I)$, relating the voltage to its terminals to the current passing through it, is a differential function with a constant coefficient. It can also be said that a dipole is linear if it has an impedance independent of the current that passes through it (impedance is the voltage/current ratio in direct mode as in sinusoidal regime). Resistors, capacitors and coils are linear dipoles. Diodes, transistors and in general semiconductor-based components are not.

- A dipole is **passive** if its voltage/current characteristic passes through the origin (if $U = 0$, $I = 0$ and vice versa). It can only consume electrical energy, and this energy is dissipated by the Joule effect

  A dipole is **active** if its characteristic does not pass through the origin and part of the energy it puts into play does not correspond to the Joule effect.

- A dipole works as a **receiver** if it **consumes** (or not) active electrical energy (joules) but **does not generate** it on average.

  A dipole works as a **source** if it provides active electrical energy.

  Ex: passive components (resistors, capacitors and coils) are necessarily receivers.

  On the other hand, the battery of your laptop is an active dipole but it can work as a source when it provides current or as a receiver when you recharge it.

  An electric motor is an active receiver: it consumes electrical power and provides mechanical power ... when it turns.

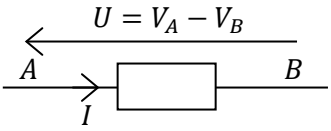
A receiving dipole is represented by a simple rectangle that can correspond either to a pure resistor or to a set of components, the capacitors and coils being represented by specific symbols that we will see later.

2. Sign conventions

Before calculating the voltage at the terminals of any dipole or the current that passes through it, we must agree on how to define the direction in which we will calculate them

a. Receiver convention

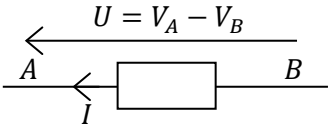
It consists in guide the voltage in the opposite direction of the current regardless of the dipole.



Consequence: if, after numeric application, the voltage and current are NOT OF THE SAME SIGN, this proves that the dipole is receiver. Indeed, in a receiver, the current flows from the highest potential point to the lowest potential point, just as water flows from top to bottom. Otherwise, it proves that the dipole is a generator (source).

b. Source convention

It consists in guiding the voltage and the current in the same direction regardless of the dipole.

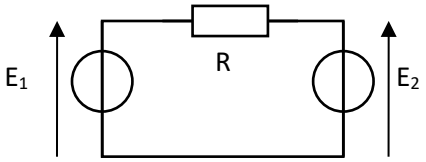


Consequence: if, after numeric application, the voltage and current are OF THE SAME SIGN, this proves that the dipole is source. Indeed, in a source, the current flows from the lowest potential point to the highest potential point, just as a pump system in a pipe (equivalent to the source) can raise the water from the bottom to the top. Otherwise, it proves that the dipole is a receiver.

Notes:

- If we know that a dipole is a source, it is in our interest to apply the source convention between the current that passes through it and the voltage at its terminals. This implies a current and a voltage with the same sign.
- If a circuit has only one source, respecting the source convention for this source (and the receiver convention for resistors) will necessarily give currents of the same sign as the voltage of the source.

Attention: in the diagram on the right, we cannot respect the source convention at the same time for E1 and for E2: it is therefore necessary to choose a random direction of the current to make the calculations and, if the numerical result is positive, this will prove that the current flows well in the chosen direction but a negative result is not false: it would only prove that the current flows in the opposite direction! (Ex: a battery that is recharged "sees" the current enter through its + terminal.



IV. Dipoles as receivers

A. The resistor

1. Resistive effect

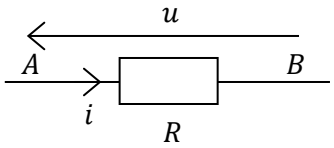
Let's consider a conductor, at whose terminals a voltage is imposed. We know therefore that a current will pass through this conductor. However, not all materials "conduct" electricity easily: some offer more or less resistance on the passage of electrons. This phenomenon is called the *resistive effect*.

2. Ohm's law

A resistor is a passive receiving dipole which, when applying a voltage between its terminals, is crossed by a current of intensity proportional to this voltage (linear response). In other words, it is a dipole where the characteristic current-voltage (the curve $I = f(U)$) is a straight line passing through the origin.

The proportion factor is called the resistance of the dipole and it is expressed in Ohms (symbol Ω), referring to the German physicist Georg-Simon Ohm who studied the laws of electrical circuits.

Symbol:



It is a receiving dipole. We therefore use the receiver convention for the indication of quantities.

The following proportionality relationship is set:

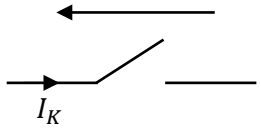
$$u(t) = R.i(t)$$

It's ***Ohm's law***.

Note 1: it only applies if the receiving convention is used; otherwise, a "-" sign must be added.

Note 2: The inverse of resistance is conductance, often denoted G. It is expressed in Siemens (S): $G = \frac{1}{R}$.

Note: The particular case of interrupter K :



An interrupter is a dipole that could be considered:

- Either as a wire when it is closed. We will have $U_K = 0$ while I_K could be any value.
- Or as an infinite resistance when it is open. We will have $I_K = 0$ while U_K could be any value.

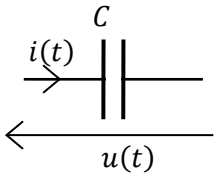
B. The capacitor

The capacitor is an electronic component than can store and then release electrical charges.

It is a dipole composed of two conductive surfaces (the plates) placed opposite each other and separated by an insulator (the dielectric). It is characterized by its capacitance C, which is expressed in Farads (F) and represents the amount of electrical charge it carries. Given the usual orders of magnitude, sub-multiples of the farad are mainly used: the microfarad ($\mu F = 10^{-6}F$), the nanofarad ($nF = 10^{-9}F$) et le picofarad ($pF = 10^{-12}F$).

Symbol:

A capacitor is represented by the following symbol:



Current/Voltage relation:

For a given circuit, its capacitance C is defined as the ratio of the accumulated charge to the voltage applied across its terminals:

$$C = \frac{q}{u}$$

Now, current is the derivative of charge per unit time. Therefore, we have:

$$i(t) = C.\frac{du}{dt}$$

Capacitors are mainly used to:

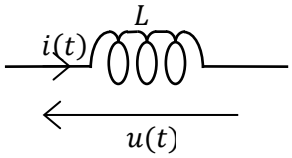
- stabilize a power supply (they discharge during voltage drops and charge during voltage spikes);
- process periodic signals (e.g., filtering)
- separate alternating current from direct current (constant current), the latter being blocked by the capacitor;
- store energy, in which case we refer to a supercapacitor.

C. The inductor (coil, self)

A coil is a common electronic component. It is a dipole consisting of a winding of conductive wire, possibly around a ferromagnetic core. It is characterized by its inductance L, which is expressed in Henry (H)

Symbol:

A coil is represented by the following symbol:



Current/Voltage relation:

You will no doubt see later in physics that the magnetic induction coefficient of the coil is defined by the ratio between the magnetic induction flux through the circuit (in short, the product of the magnetic field and the surface it encloses) and the current that gives rise to it:

$$L(t) = \frac{\phi(t)}{i(t)}$$

Furthermore, you will demonstrate in physics that the potential difference u appearing due to the self-inductive effect across the coil terminals is such that:

$$u(t) = \frac{d\phi}{dt}$$

Therfore:

$$u(t) = L.\frac{di}{dt}$$

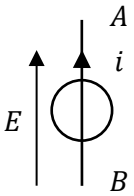
Along with capacitors and resistors, coils are one of the three passive linear components that make up electronic circuits. Coils are widely used in alternating current (AC) electronic equipment, particularly in radio equipment. They can be used to block alternating current while allowing direct current to pass through. Coils are also used in the electonic filters that we will study in the next semester.

V. Dipoles as generators (sources)

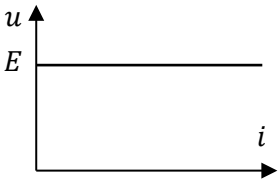
Sources are dipoles capable of sending current in a circuit but they can also function as receivers when they receive current, such as for example a battery which is being recharged. In this case, they are said to function as active receptors .

1. Ideal sources

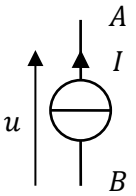
An ideal source is a dipole that supplies active power without losses due to Joule's effect. There are 2 types of these sources:



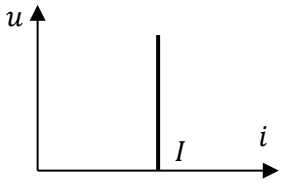
Voltage source



Curve $u = f(i)$



Current source



Curve $u = f(i)$

An ideal voltage source (cell, battery, sector) provides a voltage independent of the circuit in which it supplies (see the u = f(i) curve above).

An ideal current source (battery charger) provides a current independent of the circuit in which it flows (see the u = f(i) curve above).

Note: In continuous mode, the value of the voltage (or current) is constant over time (for example a battery). In sinusoidal regime, the effective value of the voltage (or current) is constant: for example, the voltage coming from the power sector is sinusoidal, therefore variable over time, but its effective value is constant and equal to 220 V whatever the current supplied.

The symbols of these 2 sources shown above are not drawn randomly. We will see later that it is often necessary to cancel one or more of these sources to make certain calculations. In this case, simply remove "the circle" from the symbol :

- ✓ Cancel a voltage source consists in canceling the voltage between its terminals: they are therefore at the same potential as they are linked by a single wire.
- ✓ Cancel a current source consists in canceling the current passing through the branch where it is implanted: it is clear that terminals A and B are not linked when the circle is removed. The concerned branch can be fully removed.

2. Real sources (or ohmic sources)

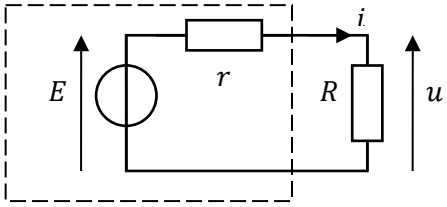
a. Voltage source

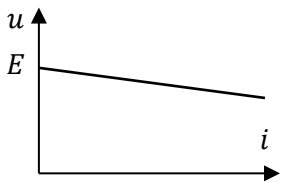
if, in the opposite diagram, $R = 0$ (R is replaced with a negligible resistance wire), then $U = 0$ which is incompatible with the equation $U = E$. The ideal model is not sufficiently realistic.



Therefore, a resistance in series is introduced with the ideal voltage source: it's called internal resistance r. It corresponds to Joule's losses in the wires connected to the source. As long as R is much greater than r, r is considered negligible compared to R and the source is admitted to be ideal and $u = E$.

The weaker *r* is, the better is the source is (closer to ideal). The output voltage u drops faster when the load resistance R weakens.





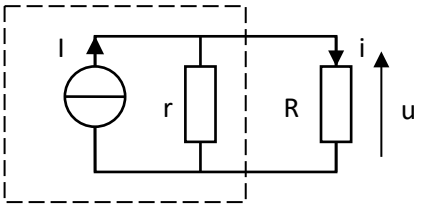
b. Current source

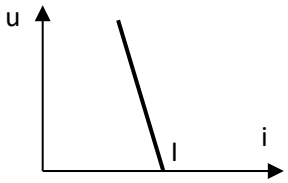
If, in the opposite diagram, $R = \infty$, (Resistance is removed), then $I = 0$ which is incompatible with I constant. The ideal model is no longer realistic.



A resistance is introduced parallel to the ideal source: it's the internal resistance r. It corresponds to losses due to Joule's effect in wire. As long as R is much less than *r*, we can neglect r in front of R and admit that the source is ideal and that i=I.

The stronger *r* is, the better is the source is (closer to ideal). The output current *i* drops faster when the load resistance R gets stronger.





B. Fundamental Laws

I. Definitions

To be able to work on electrical circuits, it is useful to define a certain number of terms and concepts:

- ✓ Node: a point serving as a junction to at least 3 conductors (wires).
- ✓ Branch: a circuit portion connecting two nodes.
- ✓ Loop: a set of branches forming a closed contour which could be traversed by passing only once through each intermediate node of the contour.

Moreover, the following concepts are agreed:

- ✓ Many dipoles are considered in series if they are passed by the same current.

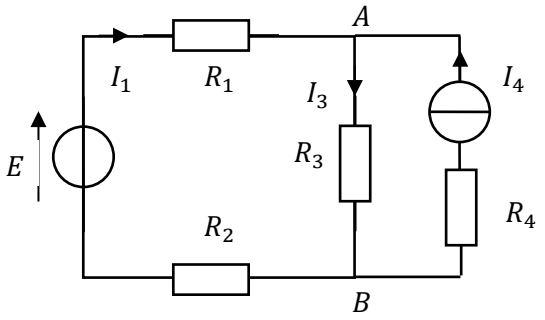
  Dipoles in series form a branch (which can contain a single dipole).
- ✓ Many dipoles are considered in parallel if the same voltage is applied between their terminals (in other words, their terminals are connected).

  Note: some branches can also be in parallel

IMPORTANT NOTE: DIPOLES ARE NOT NECESSARILY IN SERIES OR IN PARALLEL.
The only criteria to know is some dipoles are in series or in parallel is the strict application of the definitions above.

Example: the diagram on the right contains:

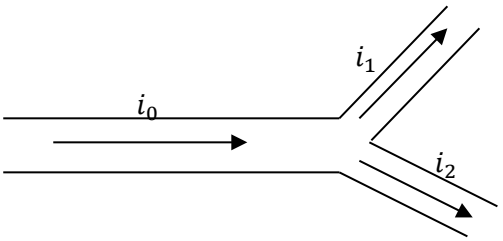
- ✓ 3 branches: (E, R_1, R_2), (R_3) and (I_4, R_4)
- ✓ 2 nodes: A and B
- ✓ 3 loops (E, R_1, R_3, R_2), (I_4, R_3, R_4) and (E, R_1, I_4, R_4, R_2)
- ✓ R_1 and R_2 are in series but R_3 is neither in series nor in parallel with R_1, R_2 or R_4.



II. Fundamental laws of electronics: Kirchhoff laws

1. Node's law

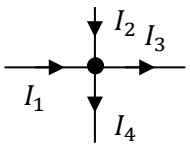
This law is easily deduced from the notion of electric current. Suppose we have a flow $i_0 = \frac{dq}{dt}$ of electrons in a conductor arriving at a node of an electrical circuit (see definition above):



Electrons coming "from the left" will leave either in the first or in the second branch. But, the total number of electrons per second will remain the same as the one that comes permanently from the left. So we have: $i_0 = i_1 + i_2$ (with the direction of the currents defined in the figure above)

We can then generalize this relationship for a node where n conductors would arrive. We then obtain the law of nodes, which states that the algebraic sum of the currents relative to a node is zero. We count positively the intensities of the currents entering the node, and negatively currents leaving it.

Ex:



Node's law applied to the diagram on the left gives:

$$I_1 + I_2 - I_3 - I_4 = 0.$$

2. Loop (mesh) law

This law stems from the remark that between any two points, the potential difference is well defined. Consider for example 3 points A, B et C. The voltages $U_{AB} = V_A - V_B$, $U_{AC} = V_A - V_C$, and $U_{CB} = V_C - V_B$ are respectively measured between A and B, A and C and B and C. The relation $U_{AB} = V_A - V_C + V_C - V_B = U_{AC} + U_{CB}$ is noticed.

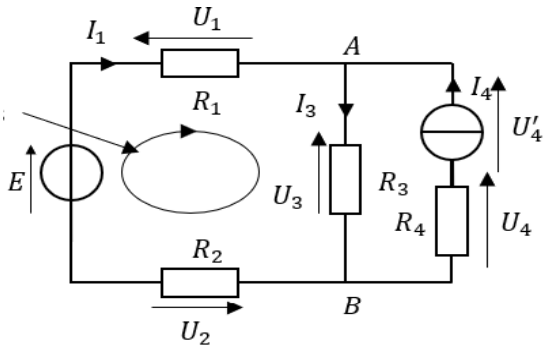
By generalizing, we obtain the law of loops which states that the algebraic sum of voltages along a **closed path** is zero.

To apply loop's law, we define an arbitrary direction along the loop and we consider it as positive. Thus, a voltage arrowed in the path direction will be preceded by the sign "+", and a voltage arrowed in the opposite direction will be preceded by the sign "-".

Ex:

- $E - U_1 - U_3 - U_2 = 0$
- $E - U_1 - U_4' - U_4 - U_2 = 0$
- $U_3 - U_4' - U_4 = 0$

Path direction

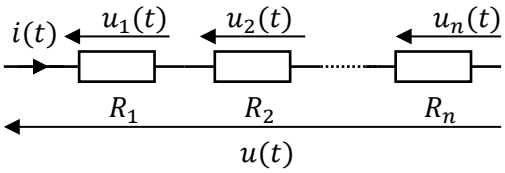


(For the previous equations, the chosen direction is always the same)

III. Resistors associations

1. Association of resistors in series

Ohm's law allows you to write: $u_k(t) = R_k . i(t)$



Loop's law allows you to write:

$$u(t) = \sum_{k=1}^{n} u_k(t) = \sum_{k=1}^{n} R_k . i(t) = i(t) . \sum_{k=1}^{n} R_k$$

since the current is the same in all resistors.

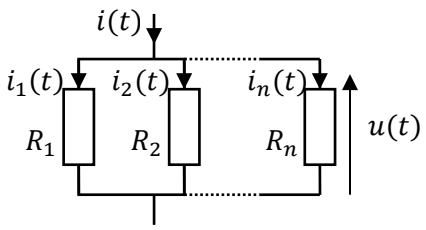
By definition, equivalent resistor R_{eq} is such that: $u(t) = R_{eq} . i(t)$, then by comparing the last 2 formulas, we obtain:

$$R_{eq} = \sum_{k=1}^{n} R_k$$

<u>Conclusion</u>: The resistor equivalent to n resistors in series is equal to their sum.

2. Association of resistors in parallel

Ohm's law allows you to write: $i_k(t) = \frac{u(t)}{R_k}$



Node's law allows you to write:

$$i(t) = \sum_{k=1}^{n} i_k(t) = \sum_{k=1}^{n} \frac{u(t)}{R_k} = u(t) . \sum_{k=1}^{n} \frac{1}{R_k}$$

because the voltage is the same at the terminals of each resistor

By definition, equivalent resistor R_{eq} is such that: $i(t) = \frac{u(t)}{R_{eq}}$, then by comparing the last 2 formulas, we obtain:

$$\frac{1}{R_{eq}} = \sum_{k=1}^{n} \frac{1}{R_k}$$

<u>Conclusion</u>: The inverse of the resistor equivalent to n resistors in parallel is equal to the sum of their inverses.

<u>Note</u>:

- ✓ If conductance is used $G = \frac{1}{R}$, we obtain: $G_{eq} = \Sigma G_i$
- ✓ For 2 resistors in parallel, it is simpler to use the following formula:

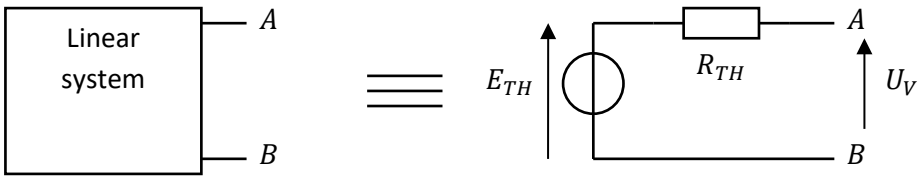
$$R_{eq} = \frac{R_1 . R_2}{R_1 + R_2}$$

IV. Thevenin and Norton theorems

3. Thevenin theorem

Any linear system with two terminals A and B can be completely characterized as an ideal voltage source (E_{TH} or Thevenin voltage) in series with one resistor (R_{TH} or Thévenin resistance, and also called internal resistance):

- ✓ E_{TH} is equal to the potential difference which appears between points A and B when they are not connected: This is the no-load voltage and can be noted as U_V.
- ✓ R_{TH} is the resistance seen from points A et B when the system is fully passive, in other words, all independent energy sources in the system are cancelled.

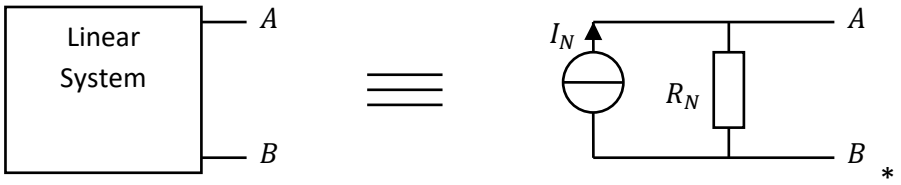


Note: Cancelling a voltage source means it should be shorted. Cancelling a current source consists of removing the branch in which it is located.

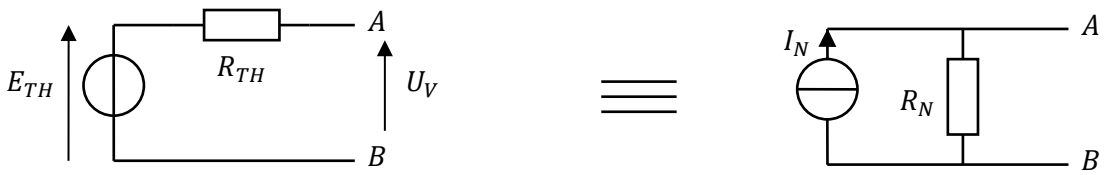
4. Norton theorem

Any linear system with two terminals A and B can be completely characterized as an ideal current source (I_N or Norton current) in parallel with one resistor (R_N or Norton resistor, and also called internal resistance):

- ✓ I_N is equal to the current which appears between points A and B when they are connected with a wire. This is short circuit current and can be noted as I_{CC}.
- ✓ R_N is the resistance seen from points A et B when the system is fully passive, in other words, when all independent energy sources are cancelled, it is the same as R_{TH}.



5. Equivalents Thevenin/Norton

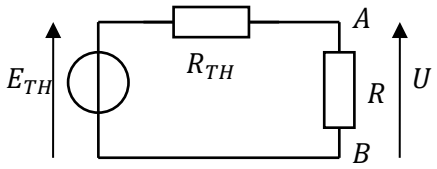


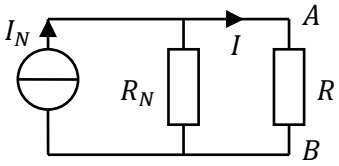
Note above circuits are equivalent to each other when:

- ✓ $E_{TH} = I_N.R_N \ \ ou\ I_N = E_{TH}/R_{TH}$
- ✓ $R_{TH} = R_N$

To go further: Difference between these two sources

Theoretically they are completely equivalent. Practically the difference is due to the order of magnitude of the internal resistance *(R_{TH} ou R_N) compared to the load resistance R.*





$$U = E_{TH}.\frac{R}{R_{TH}+R}$$

As long as $R \gg R_{TH}$, $U \approx E_{TH}\ \forall R$

$$I = I_N.\frac{R_N}{R_N+R}$$

As long as $R \ll R_N$,$I \approx I_N\ \forall R$

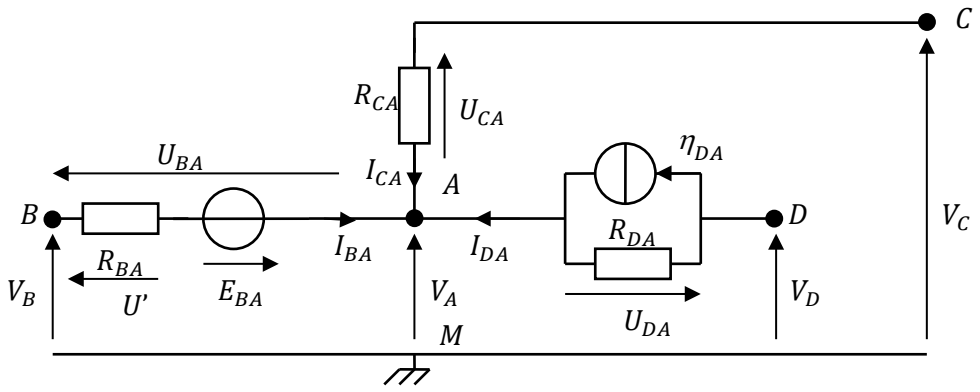
In other words, Thevenin equivalent is preferable when its internal resistance is very low and therefore a priori much lower than *R*: the output voltage U practically does not depend on *R* and remains almost equal to E_{TH} (this is the case for Operational Amplifiers).

Norton equivalent is preferable when its internal resistance is very high and therefore a priori much greater than *R*: the output current I practically does not depend on *R* and remains almost equal to I_N (this is the case for transistors).

V. Millman theorem

This theorem is a consequence of Node's law written in terms of potentials.

It is very useful for calculating the potential at a point A of the circuit, that is to say the potential difference between this point and another point M taken as a reference, all the potentials being defined in relation M (M will therefore be the 0 V level point, called "mass").



Demonstration: let's take the example in the above figure, it gathers 3 different types of sources connected with 3 different branches to node A.

Let's calculate each branch current in terms of potentials:

Branch BA: $I_{BA} = \frac{U'}{R_{BA}} = \frac{U_{BA}+E_{BA}}{R_{BA}} = \frac{V_B - V_A + E_{BA}}{R_{BA}}$

Branch CA: $I_{CA} = \frac{U_{CA}}{R_{CA}} = \frac{V_C - V_A}{R_{CA}}$

Branch DA: $I_{DA} = \eta_{DA} + \frac{U_{DA}}{R_{DA}} = \eta_{DA} + \frac{V_D - V_A}{R_{DA}}$

Then, Node's law is applied: $I_{BA} + I_{CA} + I_{DA} = 0$ as all the arrows point to A. each current is replaced with its formula in terms of potential.

$$\frac{V_B - V_A + E_{BA}}{R_{BA}} + \frac{V_C - V_A}{R_{CA}} + \eta_{DA} + \frac{V_D - V_A}{R_{DA}} = 0$$

The above equation is developed and some parts of it are factored with V_A.

$$\frac{V_B}{R_{BA}} - \frac{V_A}{R_{BA}} + \frac{E_{BA}}{R_{BA}} + \frac{V_C}{R_{CA}} - \frac{V_A}{R_{CA}} + \eta_{DA} + \frac{V_D}{R_{DA}} - \frac{V_A}{R_{DA}} = 0$$

V_A is finally expressed as:

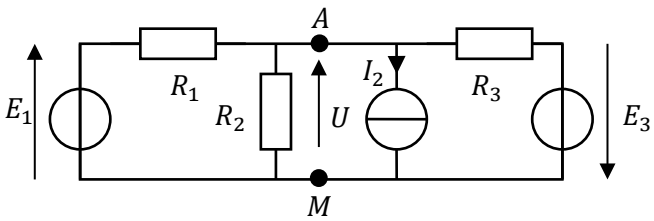
$$V_A = \frac{\frac{V_B}{R_{BA}} + \frac{E_{BA}}{R_{BA}} + \frac{V_C}{R_{CA}} + \eta_{DA} + \frac{V_D}{R_{DA}}}{\frac{1}{R_{BA}} + \frac{1}{R_{CA}} + \frac{1}{R_{DA}}}$$

The generalization of the above formula to n branches is Millman theorem:

$$V = \frac{\sum \frac{V_k}{R_k} + \sum entering\ currents - \sum exiting\ currents}{\sum \frac{1}{R_k}}$$

Example:

Le circuit on the right seems relatively complex, we want to determine the expression of voltage $U\ (=\ V_A - V_M)$.



Superposition theorem can be used: 3 voltages should be calculated keeping only one source at a time, then add the calculated voltages.

Thevenin or Norton equivalents can be used, however, calculations will be long and fastidious: R_{TH} et E_{TH} must be calculated and it does not see, easy or straightforward, then U is calculated using voltage divider formula.

Looking at the diagram, we notice that all voltages (E_1, E_3 and U) have a common point materialized with the horizontal line in the bottom of the figure. This point is considered as a reference point for voltages, in other words, its voltage level is 0 Volts (like the point M in the demonstration of Millman theorem).

Branch (E_1, R_1) corresponds to the branch *BA* on the left in the precedent diagram under the hypothesis $V_B=0$.

Branch (E_3, R_3) corresponds to the branch *CA* on the top in the precedent diagram under the hypothesis $V_C=0$.

Branch (I_2, R_2) corresponds to the branch *DA* on the in the precedent diagram under the hypothesis $V_D=0$.

Important precision: in Millman theorem demonstration, voltage and currents are preceded by the sign "+" as they are “oriented” towards point A whose potential is foreseen. If one of the voltages or currents in the branches connected to point A is oriented in the opposite direction (arrow directed towards M), then its value must be preceded by sign "-".

Taking into account this precision and applying Millman theorem on the diagram above, we can directly write:

$$U = \frac{\frac{E_1}{R_1} - \frac{E_1}{R_3} - I_2 + \frac{0}{R_2}}{\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2}} = R_2 . \frac{(E_1 . R_3 - E_3 . R_1 - I_2 . R_1 . R_3)}{R_2 . R_3 + R_1 . R_2 + R_1 . R_3}$$

We notice that only a single formula in sufficient to calculate *U*.

C. Sinusoidal steady state - Definitions

I. Sinusoidal currents and voltages

1. Definition

A current or voltage is said to have a sinusoidal waveform if their amplitudes vary according to a sine law.

2. Notation

$s(t) = S.\sqrt{2}.\cos(\omega.t + \varphi)$ for a sinusoidal signal (current or voltage).

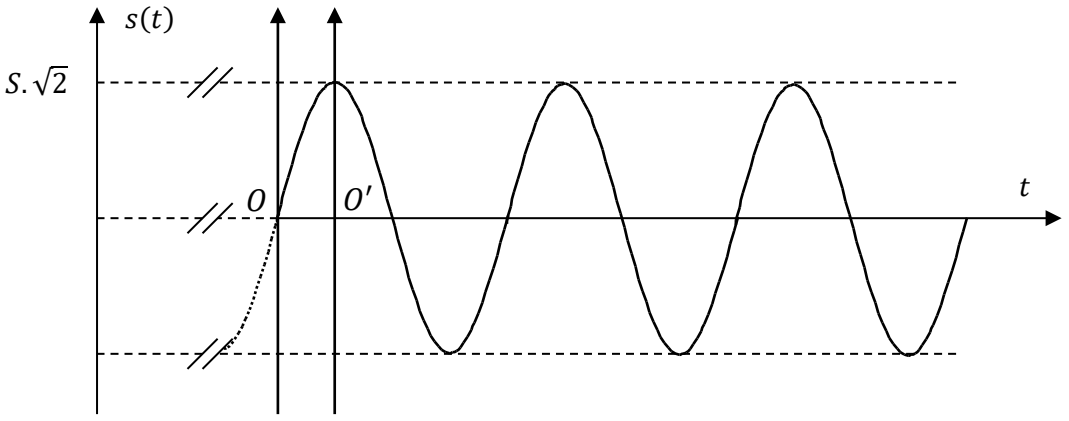
Note: We would have the same definitions if $s(t) = S.\sqrt{2}.\sin(\omega.t + \varphi)$

- $s(t)$ is the instantaneous value: it varies over time and is denoted in lower case.
- S is the RMS value of $s(t)$ (Root Mean Square). It is therefore necessarily constant, real and positive. As function cosine or sine has no unit, S will have the same unit as the signal it represents (Ampere if s(t) is a current, Volt if it is a voltage).
- ω is the angular velocity or pulsation and is expressed in radians per second (rads/s).

Since the radian is dimensionless (it is a quotient of 2 lengths), ω is inversely homogeneous to time exactly like a frequency f. These 2 quantities express the same idea of angular velocity, ω in number of radians per second and f in number of "revolutions" per second. They are therefore proportional. Thus, if, in 1 second, the angle varies by ω radians, then it will vary by 2π over a period T.
This leads to $\omega.T = 2.\pi$, i.e., $\omega = \frac{2.\pi}{T} = 2.\pi.f$

- φ is the phase at the origin, i.e. for t = 0. It is expressed in radians (or degrees).

But we must understand an important point: the sinusoidal regime is a so-called "established" regime: the quantities have been sinusoidal for a theoretically infinite time. On a graph representing a sinusoidal quantity, one might not present the origin of time; it is done so that the drawing is more readable.
If we position the origin of the times (the vertical axis) at an instant where $s(t) = 0$, then $s(t) = S.\sqrt{2}.\sin(\omega t)$ (as $\sin(\omega t) = 0$ for $t = 0$). If time origin (the vertical axis) at a time where $s(t) = S.\sqrt{2}$, then $s(t) = S.\sqrt{2}.\cos(\omega t)$ (as $\cos(\omega t) = 1$ for $t = 0$).



We see on this graph that the **same sinusoid** can be denoted $S.\sqrt{2}.\sin(\omega.t)$ if time origin is O and $S.\sqrt{2}.\cos(\omega.t)$ if the time origin is O'.

In sinusoidal regime, what we should look for is the delay or advance of one magnitude with respect to another, arbitrarily taken as the phase reference and for which φ is zero by definition.

Analog example: 3 cars A, B, C turn on a circular track, at the same speed for an indefinite time. If we choose A as a reference, we will for example have B 20° behind (compared to A) and C ahead of 45° (always with respect to A). But we can also choose B as a reference and in this case A will be 20° ahead (with respect to B) and C will be 65° ahead (with respect to B).

In electronics, the voltage at the terminals of a capacitor is 90° behind the current that crosses it if we choose the current as the phase reference. In other words, the voltage lags the current by 90°. The current in a capacitor is 90° ahead of the voltage at its terminals if we choose the voltage as the phase reference. In other words, the current leads the voltage by 90°.

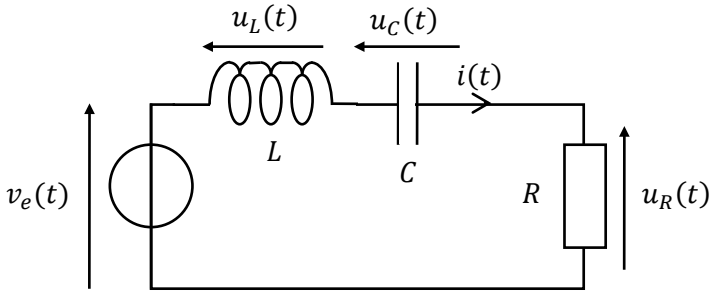
The rule is therefore to always use the same trigonometric function (either sine or cosine), never mix the 2 notations, and to arbitrarily choose a signal as the phase reference: the values of φ of the other signals will indicate their phase shift with respect to that taken as phase reference.

Important: If a linear system is controlled by a sinusoidal signal with pulsation ω then all the signals in the system are sinusoidal with the same pulsation ω.

II. Example of a circuit in steady sinusoidal regime: Study of a RLC circuit

Consider the diagram on the opposite where $v_e(t) = V_E.\sqrt{2}.\sin(\omega.t)$.

Let's find the expression of the current $i(t)$.



Loop's law give:

$$v_e(t) - u_L(t) - u_C(t) - u_R(t) = 0$$

In addition, we know that:

$$\begin{cases} u_L(t) = L.\dfrac{di(t)}{dt} \\ u_R(t) = R.i(t) \\ i(t) = C.\dfrac{du_C(t)}{dt} \end{cases}$$

We thus obtain:

$$v_e(t) = L.\frac{di(t)}{dt} + R.i(t) + \frac{1}{C}.\int i(t).dt$$

Deriving the above equation gives:

$$\frac{dv_e(t)}{dt} = L.\frac{d^2i(t)}{dt^2} + R.\frac{di(t)}{dt} + \frac{i(t)}{C}$$

We thus obtain a differential equation of the 2nd order, with constant coefficient, with second sinusoidal member... that I let you solve …
A better solution to such a problem is to use complex representation. Each element of the circuit (signal and component) will be associated with a complex number defined in the next paragraph.

3. Complex representation

1. Complex amplitude

It is the complex associated with a sinusoidal signal (current or voltage).

Definition : Consider $s(t) = S.\sqrt{2}.\cos(\omega.t + \varphi)$.

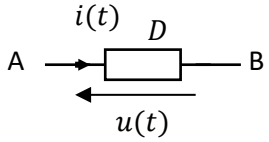
The complex amplitude associated with the signal $s(t)$ is the complex $\underline{S}$ such that:

- $|\underline{S}| = S$**, RMS value of the signal** $s(t)$
- $\arg(\underline{S}) = \varphi$**, phase at the origin of** $s(t)$

Note: the definition is the same if $s(t) = S.\sqrt{2}.sin(\omega.t + \varphi)$

2. Complex impedance

It's the complex associated with a dipole.



Def: Consider a passive linear dipole D where current and voltage are arrowed respecting the receiver convention.

Let's set: $\begin{cases} u(t) = U.\sqrt{2}.\cos(\omega.t + \varphi_u) \\ i(t) = I.\sqrt{2}.\cos(\omega.t + \varphi_i) \end{cases}$

The complex impedance of this dipole is the quantity $\boxed{\underline{Z} = \frac{\underline{U}}{\underline{I}}}$

where $\underline{U}$ et $\underline{I}$ are the complex amplitudes of the sinusoidal signals $u(t)$ and $i(t)$.

Notes:

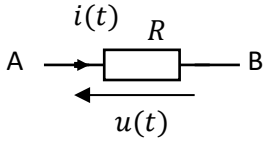
1. $|\underline{Z}| = \frac{|\underline{U}|}{|\underline{I}|} = \frac{U}{I}$: this quantity is expressed in $\frac{V}{A}$, i.e., in Ω (Ohm).
2. $\arg(\underline{Z}) = \arg(\underline{U}) - \arg(\underline{I}) = \varphi_u - \varphi_i$: it's the phase shift of the voltage $u(t)$ at dipole terminals with respect to the current $i(t)$ crossing the dipole.

$\underline{Z} = \frac{\underline{U}}{\underline{I}} \Leftrightarrow \underline{U} = \underline{Z}.\underline{I}$: *The interest of this notion is to obtain a relationship between complex quantities analogue to Ohm's law*.

Impedances of usual dipoles

a) Resistor

From the relation voltage/current $u = Ri$, we deduce that:

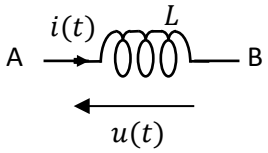


$$\boxed{\underline{Z_R} = R}$$

For a resistor, the voltage and the current, expressed according to the receiver convention, are in phase.

b) Inductor (coil)

The characteristic voltage/current relation of a pure inductor is $u(t) = L\frac{di}{dt}$, which translate into complex quantities as $\underline{U} = jL\omega\underline{I}$. The complex impedance of a pure inductor is therefore:

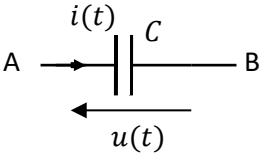


$$\boxed{\underline{Z_L} = jL\omega}$$

The voltage leads the current by $\pi/2$ with the receiver convention.

c) Capacitor

In the same way as before, the characteristic current/voltage relation of a capacitor is $i(t) = C.\frac{du}{dt}$, which translate into complex quantities as $\underline{I} = jC\omega\underline{U}$. The complex impedance of a capacitor is therefore:



$$\boxed{\underline{Z_C} = \frac{1}{jC\omega}}$$

Acoording to the receiver concention, the current leads the voltage by $\pi/2$.

Associating elements

Given the analogy between resistance for real values and impedance for complex values, we will observe similar laws of association for complex impedances.

a) In series

The complex impdance resulting of the association in series of linear dipoles is the sum of the complex impedance of each dipole.

b) In parallel

The complex admittance (inverse of the complex impedance) of the association in parallel of linear dipoles is the sum of complex admittance of each dipole.

Linear networks in sinusoidal steady state

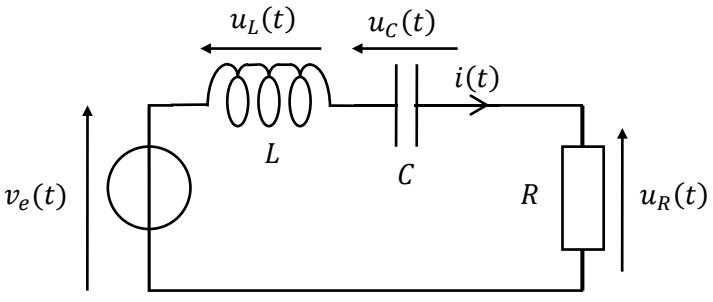
The laws related to the study of linear networks, previously seen, remain valid in sinusoidal steady state, provided all the sources operate at the same frequency and that the steady state is reached.

Under the above conditions, the laws are applied using the complex quantities.

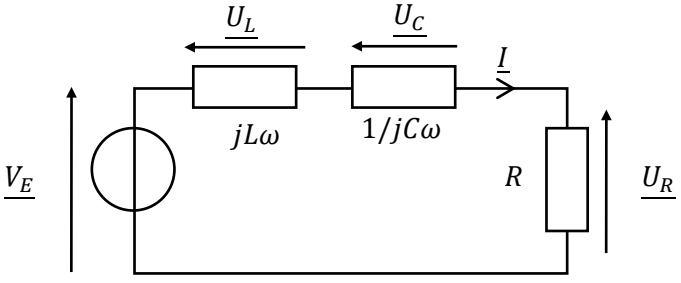
In particular, Thévenin and Norton equivalents are generalized, by replacing the term resistance by complex impedance.
Similarly, Millman's theorem, voltage and current dividers are transposed the same way.

Example: RLC series circuit

Let's go back to the previous circuit. In order to determine the expression of $i(t)$, it was necessary to solve a differential equation of the 2nd order including a second member.
Let's use the complex representation to find the expression of $i(t)$:



1st step: We move into complex representation. To do this, each element is replaced by its associated complex. (Complex impedance for components and complex amplitude for currents and voltages).



Note: You might notice that the symbols of capacitors and inductors have been replaced by a symbol similar to that of a resistor. This is justified by the fact that the current/voltage relation in complex is similar to Ohm's law.

2nd step: We use the laws and theorems of electronics to find the expression of the complex amplitude $\underline{I}$ associated to the sinusoidal current $i(t)$. Here, loop's law is used:

$$\underline{V_E} - \underline{U_L} - \underline{U_C} - \underline{U_R} = 0$$

We also know that:

$$\begin{cases} \underline{U_L} = jL\omega.\underline{I} \\ \underline{U_C} = \dfrac{1}{jC\omega}.\underline{I} \\ \underline{U_R} = R.\underline{I} \end{cases}$$

Therefore:

$$jL\omega.\underline{I} + \frac{1}{jC\omega}.\underline{I} + R.\underline{I} = \underline{V_E}$$

Finally:

$$\underline{I} = \frac{\underline{V_E}}{jL\omega + \frac{1}{jC\omega} + R} = \frac{jC\omega}{1 + jL\omega.jC\omega + R.jC\omega}.\underline{V_E}$$

$$\boxed{\underline{I} = \frac{jC\omega}{1 - LC\omega^2 + jRC\omega}.\underline{V_E}} \quad (j^2 = -1)$$

3rd step: the temporal expression of sinusoidal current $i(t)$ is calculated.

The above diagram is linear and $v_e(t) = V_E.\sqrt{2}.\sin(\omega.t)$ (cf p.3), then the current can be expressed as $i(t) = I.\sqrt{2}.\sin(\omega.t + \varphi)$ with:

- $I = \left|\underline{I}\right| = \frac{|jC\omega|}{|1-LC\omega^2+jRC\omega|}.\left|\underline{V_E}\right| = \frac{C\omega}{\sqrt{(1-LC\omega^2)^2+(RC\omega)^2}}.V_E$
- $\varphi = \arg(\underline{I}) = \arg(jC\omega) + \arg\left(\underline{V_E}\right) - \arg(1 - LC\omega^2 + jRC\omega)$

$$= \begin{cases} \frac{\pi}{2} + 0 + \arctan\left(\frac{RC\omega}{1-LC\omega^2}\right) & si\ 1 - LC\omega^2 > 0 \\ \frac{\pi}{2} + 0 + \pi - \arctan\left(\frac{RC\omega}{1-LC\omega^2}\right) & si\ 1 - LC\omega^2 < 0 \end{cases}$$